Cauchy problem in spacetimes with closed timelike curves

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The laws of physics might permit the existence, in the real Universe, of closed timelike curves (CTC's). Macroscopic CTC's might be a semiclassical consequence of Planck-scale, quantum gravitational, Lorentzian foam, if such foam exists. If CTC's are permitted, then the semiclassical laws of physics (the laws with gravity classical and other fields quantized or classical) should be augmented by a principle of self-consistency, which states that a local solution to the equations of physics can occur in the real Universe only if it can be extended to be part of a global solution, one which is well defined throughout the (nonsingular regions of) classical spacetime. The consequences of this principle are explored for the Cauchy problem of the evolution of a classical, massless scalar field Φ (satisfying $\Box\Phi=0$) in several model spacetimes with CTC's. In general, self-consistency constrains the initial data for the field Φ. For a family of spacetimes with traversible wormholes, which initially possess no CTC's and then evolve them to the future of a stable Cauchy horizon $\mathcal{H}$, self-consistency seems to place no constraints on initial data for Φ that are posed on past null infinity, and none on data posed on spacelike slices which precede $\mathcal{H}$. By contrast, initial data posed in the future of $\mathcal{H}$, where the CTC's reside, are constrained; but the constraints appear to be mild in the sense that in some neighborhood of every event one is free to specify initial data arbitrarily, with the initial data elsewhere being adjusted to guarantee self-consistent evolution. A spacetime whose self-consistency constraints have this property is defined to be "benign with respect to the scalar field Φ." The question is posed as to whether benign spacetimes in some sense form a generic subset of all spacetimes with CTC's. It is shown that in the set of flat, spatially and temporally closed, 2-dimensional spacetimes the benign ones are not generic. However, it seems likely that every 4-dimensional, asymptotically flat space-time that is stable and has a topology of the form $R\times(S-\text{one point})$, where S is a closed 3-manifold, is benign. Wormhole spacetimes are of this type, with $S=S^1\times S^2$. We suspect that these types of self-consistency behavior of the scalar field Φ are typical for noninteracting (linearly superposing), classical fields. However, interacting classical systems can behave quite differently, as is demonstrated by a study of the motion of a hard-sphere billiard ball in a wormhole spacetime with closed timelike curves: If the ball is classical, then some choices of initial data (some values of the ball's initial position and velocity) give rise to unique, self-consistent motions of the ball; other choices produce two different self-consistent motions; and others might (but we are not yet sure) produce no self-consistent motions whatsoever. By contrast, in a path-integral formulation of the nonrelativistic quantum mechanics of such a billiard ball, there appears to be a unique, self-consistent set of probabilities for the outcomes of all measurements. This paper's conclusion, that CTC's may not be as nasty as people have assumed, is reinforced by the fact that they do not affect Gauss's theorem and thus do not affect the derivation of global conservation laws from differential ones. The standard conservation laws remain valid globally, and in asymptotically flat, wormhole spacetimes they retain a natural, quasilocal interpretation.

I. INTRODUCTION AND OVERVIEW

Physicists typically have been reluctant to consider the possibility that our universe might possess closed timelike curves (CTC's). The science-fiction spectre of a human going back in time and killing his younger self has made it seem that CTC's necessarily entail unacceptable violations of causality, and thus must be forbidden by the laws of physics.

Recently three of the authors (Morris, Thorne, and

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Yurtsever[1]) discovered that, in order to forbid CTC's, the laws might also have to forbid the existence of traversible wormholes. This is because by moving one mouth of such a wormhole at high speed away from the other and then bringing it back ("twins-paradox motion"), one can change the manner in which time hooks up through the wormhole and thereby create CTC's.[1-3] In fact, generic relative motions of the wormhole mouths will produce CTC's,[1,2] as will generic gravitational redshifts at the wormhole mouths due to generic external gravitational fields.[3] At least this is so if the Cauchy horizon at which the CTC's arise is stable.

It seems highly likely that the Cauchy horizon *is* stable against classical perturbations,[1] but stability against quantum-field perturbations is less certain: it might turn out that vacuum fluctuations of quantum fields generically produce a divergent renormalized stress-energy at the Cauchy horizon, thereby preventing the creation of CTC's.[4] However, in this paper we shall not attempt to treat quantum fields. Instead, we shall focus attention on classical physics and first-quantized quantum physics in spacetimes with CTC's.

The conventional reaction to the possibility that generic motions of traversible wormholes produce CTC's is to assert that the laws of physics must forbid traversible wormholes. We consider this a definite possibility, and elsewhere[1] we have discussed two ways in which it might come about: (i) Quantum field theory in a wormhole spacetime[5] might enforce either the averaged weak energy condition (a condition which, we have shown, must be violated in or near the throat of any traversible wormhole), or perhaps some other "weaker" energy condition (which also might necessarily be violated in a traversible wormhole);[6] (ii) our Universe might have been created without any traversible wormholes, and the laws of physics might prevent the topology change that accompanies the construction of such wormholes when initially there are none.

It is far from clear whether either of these possibilities is true, and to find out is likely to involve extensive, long research efforts. Accordingly, we think it appropriate to explore the consequences of an alternative possibility: that the laws of physics do not rule out traversible wormholes, and that the Cauchy horizons which form as a result of such wormholes' motions are stable against vacuum fluctuations and therefore give rise to CTC's.

This possibility is made all the more intriguing by the fact that some of the current efforts to understand quantum gravity involve nontrivial topologies, i.e., "quantum foam," on scales of the Planck length.[7] In its present incarnation, this quantum foam entails finite probability amplitudes for Planck-scale, "Euclidean wormholes," i.e., wormholes with positive-definite four-dimensional metrics, which link widely separated regions of space. By contrast, when originally conceived by Wheeler[8] in the 1950s, quantum foam entailed finite probability amplitudes for Planck-scale "Lorentzian wormholes," i.e., submicroscopic versions of the classical, spacetime wormholes treated in this paper. The change from Lorentzian to Euclidean foam[9] was motivated by the belief that a Lorentzian path integral would not converge, as well as by an aversion to the closed timelike curves that accompany topology change in Lorentzian spacetimes.

The manner in which the Euclidean quantum foam gives rise to classical spacetime on scales large compared to the Planck length is not yet fully understood. An important aspect of this is the fate of the foam's Euclidean wormholes. Are they wholly an artifact of the quantum regime, an artifact that goes away completely in the classical limit? Or can they give rise to Lorentzian wormholes with CTC's when the quantum-to-classical transition is made? Even before the transition, can the quantum foam itself be regarded in any meaningful sense as made from Lorentzian wormholes as well as or instead of from Euclidean wormholes?

It is tempting to believe that the semiclassical laws of physics inherit their attitude toward traversible wormholes and CTC's from quantum gravity's answers to these questions. If quantum gravity allows Lorentzian foam and Lorentzian wormholes in its quantum-to-classical transition, then it thereby might force on the semiclassical laws a positive attitude toward traversible wormholes and CTC's. If quantum wormholes are entirely an artifact of Euclidean quantum gravity and disappear entirely when one enters the Lorentzian domain, then quantum gravity might force the semiclassical laws to forbid the existence of macroscopic, traversible wormholes with CTC's. As its means of enforcement, (i) quantum gravity might insist that all nongravitational quantum fields in a classical wormhole spacetime obey some energy condition (e.g., the averaged weak energy condition) that forbids the wormhole to be traversible, or (ii) quantum gravity might cause vacuum fluctuations of quantum fields to produce divergent stress-energy whenever a classical traversible wormhole tries to generate CTC's.

These speculations suggest that by exploring the possibility of incorporating traversible wormholes and CTC's into semiclassical physics, we might be probing quantum gravity's attitudes toward Lorentzian foam and Lorentzian wormholes.

One way to explore semiclassical physics' attitude toward CTC's is to inquire whether CTC's can be incorporated into the semiclassical laws of physics without producing unacceptable causality violations. The answer to this question depends, of course, on one's definition of "unacceptable."

The only type of causality violation that the authors would find unacceptable is that embodied in the science-fiction concept of going backward in time and killing one's younger self ("changing the past"). Some years ago one of us (Novikov[10]) briefly considered the possibility that CTC's might exist and argued that they cannot entail this type of causality violation: Events on a CTC are already guaranteed to be self-consistent, Novikov argued; they influence each other around the closed curve in a self-adjusted, cyclical, self-consistent way. The other authors recently have arrived at the same viewpoint.

We shall embody this viewpoint in a *principle of self-consistency,* which states that *the only solutions to the laws of physics that can occur locally in the real Universe*

are those which are globally self-consistent. This principle allows one to build a local solution to the equations of physics only if that local solution can be extended to be part of a (not necessarily unique) global solution, which is well defined throughout the nonsingular regions of the spacetime.

This principle is a conjectured property of the semiclassical limit (classical gravity; other fields classical or quantum) of the ultimate "theory of everything." In applying the principle, one is expected, at least conceptually, to allow for the gravitational back action of the nongravitational fields on the spacetime's curvature via the semiclassical Einstein field equations. The back action will become especially important if the CTC's cause the (renormalized expectation value of the nongravitational fields') stress-energy tensor to grow large. Otherwise one may be able to ignore back action—and in this paper we shall do so.

That the principle of self-consistency is not totally tautological becomes clear when one considers the following alternative: The laws of physics might permit CTC's; and when CTC's occur, they might trigger new kinds of local physics which we have not previously met. For example, a quantum-mechanical system, propagating around CTC's, might return to where it started with values for its wave function Ψ that are inconsistent with the initial values; and it might then continue propagating and return once again with a third set of values, then a fourth, then a fifth, ... The result would be a many-valued wave function (which, of course, is forbidden by standard local physics); and the wave function's many values might participate, through some new and as-yet-unknown physical law, in determining the outcomes of physical measurements. The principle of self-consistency is intended to rule out such behavior. It insists that local physics is governed by the same types of physical laws as we deal with in the absence of CTC's: laws that entail self-consistent single valuedness for the fields. In essence, the principle of self-consistency is a principle of no new physics. If one is inclined from the outset to ignore or discount the possibility of new physics, then one will regard self-consistency as a trivial principle.

The principle of self-consistency by fiat forbids changing the past. However, it does so at the price of constraining the inital data for the Cauchy problem. The main goal of this paper is to study those constraints.

For simplicity, we shall focus attention primarily on the Cauchy problem for a massless, classical scalar field Φ which satisfies the wave equation without curvature coupling, $\Box\Phi=0$. This is a severe restriction. It is quite possible that the constraints of self-consistency will have rather different characters for quantized fields and nonlinear fields from that which we shall meet for the classical, linear field Φ. It will be important, in future research, to explore the effects of quantization and nonlinearity. As a tool for such exploration, in Secs. II G and II H we shall pose and briefly discuss a set of model problems for colliding billiard balls, both classical and quantum mechanical.

In this paper we shall study the Cauchy problem for the field Φ in several model spacetimes with CTC's. Our chosen spacetimes fall into two classes: asymptotically flat spacetimes with wormholes (Sec. II); and perfectly flat spacetimes with toroidal topologies (Sec. III). These classes are broad enough to exhibit a variety of different behaviors: In some cases self-consistency seems to place no constraints whatsoever on initial data for the field Φ. In other cases there will be one simple integral constraint on the data. In others there will be periodicity constraints. And in some cases the constraints will be so severe that there will exist no nonconstant solutions whatsoever to $\Box\Phi=0$.

Some of our examples involve asymptotically flat wormhole spacetimes in which there initially are no CTC's, but which evolve CTC's to the future of a stable Cauchy horizon $\mathcal{H}$. In these spacetimes we argue (see Sec. II C below and Ref. 11) that self-consistency places no constraints whatsoever on initial data that are posed *before* $\mathcal{H}$: The standard initial data (values of Φ and $\partial\Phi/\partial t$ on an initial spacelike hypersurface of constant t to the past of $\mathcal{H}$ or values of $r\Phi$ on $\mathcal{I}^-$) are just what is needed to produce a unique solution of $\Box\Phi=0$ throughout the spacetime, including the future of $\mathcal{H}$. Two of the authors (Friedman and Morris[11]) have actually proved that this is the case for a simple example of such a spacetime. We conjecture that this well-posed nature of the initial-value problem is true for initial data on a spacelike hypersurface (but not necessarily of data on $\mathcal{I}^-$) not only for our specific wormhole spacetimes, but also for any 4-dimensional, asymptotically flat, classical spacetime with topology of the form $R\times(S-\text{one point})$, where S is a closed 3-manifold; and we also conjecture that in such spacetimes the initial-value problem is well posed, in the same sense as for Φ, for all other noninteracting (linearly superposing) fields, classical and quantum. (Note: wormhole spacetimes are of this type with $S=S^1\times S^2$.)

Although initial data posed to the past of $\mathcal{H}$ are unconstrained in these wormhole spacetimes, those posed on hypersurfaces in the future of $\mathcal{H}$ are significantly constrained. It seems to us that these constraints are best viewed as arising *not* from the existence of CTC's (after all, data given freely in the past of $\mathcal{H}$ must also deal with the CTC's), but rather from the nonexistence of reasonable global spacelike hypersurfaces in the future of $\mathcal{H}$ on which to pose initial data. The character of the constraints supports this viewpoint.

The constraints on initial data posed in the future of $\mathcal{H}$ appear to be mild in a sense that was discovered in the geometric optics limit by Novikov and Thorne (Sec. II E) and that has been generalized, made precise, and explored in detail by Yurtsever.[12] Roughly speaking, the constraints are global, but not local. More specifically, consider any event $\mathcal{P}$ to the future of $\mathcal{H}$ and any spacelike hypersurface $\mathcal{S}$ through $\mathcal{P}$. Then there exists a neighborhood of $\mathcal{P}$ in $\mathcal{S}$ in which one can specify freely Φ and $\partial\Phi/\partial t$. (Here $\partial/\partial t$ is the normal to $\mathcal{S}$.) No matter how these data are chosen, the initial data elsewhere on $\mathcal{S}$ can always be adjusted in such a way as to preserve self-consistency of the evolved field Φ.

Yurtsever[12] terms *benign for the field* Φ any spacetime in which the self-consistency constraints have this prop-

erty at all events $\mathcal{P}$. We sketch a proof, in Sec. II E, that in the geometric optics limit our wormhole spacetimes are benign for the field Φ; and it seems highly likely that they are also benign when one permits Φ to contain all wavelengths, not just the short ones of geometric optics. The structure of our geometric-optics proof makes it seem plausible that the class of spacetimes in which we expect the Cauchy problem for Φ to be well posed before the Cauchy horizon $\mathcal{H}$ [those with topology $R \times (S - \text{one point})$ where S is a closed 3-manifold] will also be benign after $\mathcal{H}$, providing they are stable for the evolution of Φ.

By contrast, we shall see in Sec. III that flat, 2-dimensional, spatially and temporally closed (i.e., toroidal) spacetimes are typically not benign. This is one more warning (see Ref. 1 for others) that 2-dimensional model spacetimes are not good guides to the CTC behaviors of 4-dimensional spacetimes.

The implications of the principle of self-consistency for the field Φ might be strongly influenced by the fact that Φ is a noninteracting (linearly superposing) field. More likely to produce peculiar results is a system that, after traveling around a nearly closed timelike world line, can interact with its younger self (e.g., a person who tries to kill his younger self). The simplest such classical system is a hard-sphere billiard ball. It turns out[13] (Sec. II G) that standard initial data (initial position and velocity) for such a billiard ball do not necessarily produce a unique, self-consistent billiard-ball evolution. Rather, some choices of initial data, posed before the stable Cauchy horizon of a wormhole spacetime, produce a unique, self-consistent evolution. Other choices produce two self-consistent evolutions (e.g., one with no passage through the wormhole and thus no self-collision, and one with wormhole passage and self-collision). And it may be (but we are not yet sure) that other choices of initial data are unable to produce any self-consistent, classical evolutions.

If physics were classical at heart, rather than quantum mechanical, these disturbing results might be enough to make us hope that the laws of physics forbid CTC's. However, when treated quantum mechanically the self-colliding billiard ball may well behave more reasonably than when treated classically[14] (Sec. II H).

The simplest way to impose the principle of self-consistency in quantum mechanics (in a classical spacetime) is by a sum-over-histories formulation in which one includes all those, and only those, histories that are self-consistent. It turns out that,[14] at least formally (modulo such issues as the convergence of the sum), for every choice of the billiard ball's initial, nonrelativistic wave function before the Cauchy horizon, such a sum over histories produces unique, self-consistent probabilities for the outcomes of all sets of subsequent measurements. When the initial wave function is a wave packet that imitates classical initial data for which there are two classical evolutions, a WKB approximation to the sum over histories gives unique, finite probability amplitudes for measurements that test for each of the classical evolutions;[14] see Sec. II H.

We suspect, more generally, that for any quantum system in a classical wormhole spacetime with a stable Cauchy horizon, the sum over all self-consistent histories will give unique, self-consistent probabilities for the outcomes of all sets of measurements that one might choose to make.

A recurring theme in this paper is the conclusion that CTC's may not be as nasty as people have assumed. This conclusion is reinforced, in Sec. II F, by an analysis of global conservation laws in asymptotically flat wormhole spacetimes with CTC's. It is shown that, just as without CTC's so also with them, all differential conservation laws expressed as the vanishing divergence of a vector field (e.g., the conservation of charge and baryons, and in stationary spacetimes the conservation of energy) give rise to global conservation laws. When CTC's are present, these global laws express the constancy of the the sum of three quantities: (i) the amount of the given item that is present in the Universe at a specific moment of time, plus (ii) the amount that is temporarily absent by virtue of time traveling to the future through a wormhole, minus (iii) the amount that is temporarily doubly present because it time traveled to the past through the wormhole. It seems reasonable to hope that, in wormhole spacetimes with CTC's, the sum-over-histories formulation of quantum mechanics will give rise to this type of conservation law for probabilities (Sec. II H). However, thus far we have not been able to verify that this is so.[14]

The remainder of this paper is divided into three sections: Section II studies the Cauchy problem for $\Box\Phi=0$ and for self-colliding billiard balls in asymptotically flat, wormhole spacetimes; Sec. III studies the Cauchy problem for $\Box\Phi=0$ in flat, 2-dimensional spacetimes with toroidal topologies; and Sec. IV gives concluding remarks and a few speculations about whether and how self-consistency constrains free will.

Throughout this paper we shall use the notational conventions of Misner, Thorne, and Wheeler[15] (MTW), including geometrized units (Newton's gravitation constant G and speed of light c set equal to unity).

II. ASYMPTOTICALLY FLAT SPACETIMES WITH WORMHOLES

A. Foundations

In this section we shall study 4-dimensional spacetimes that are empty and flat (globally Minkowskian) except for (i) a single wormhole (Fig. 1), (ii) enough additional "material" to force the wormhole's mouths to move through the external spacetime along specified, accelerated world lines, and (iii) the scalar field Φ whose evolution we shall study. The demand that the wormhole be traversible guarantees quite generally (independently of any symmetries) that it must be threaded by fields in quantum states that violate the averaged weak energy condition.[1] We shall presume, for simplicity, that our scalar field Φ does not interact with those quantum fields (except gravitationally), and that Φ is sufficiently weak that its own stress-energy does not influence significantly the curvature of spacetime.

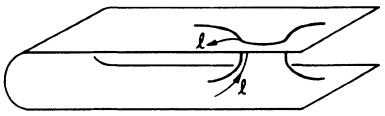


FIG. 1. Embedding diagram for a wormhole connecting two regions of a flat, Minkowski spacetime.

In the special case of a static, spherical wormhole the general-relativistic theory of the wormhole structure is particularly simple; see Ref. 16 for details. In Ref. 1 we show that the mouths of the wormhole can move through the external spacetime in arbitrary manners without any significant change of the wormhole's internal structure, so long as the accelerations g of the mouths are small compared to $1/$(their radii b): $g \ll 1/b$.

In this paper, for conceptual and computational ease, we shall use a particularly simple model in which the wormhole's length is arbitrarily small. This model (which is a simple generalization to accelerated motion of a model previously developed by Friedman *et al.*[17]) can be constructed as follows; see Fig. 2. In Minkowskii spacetime choose two timelike world lines (labeled $A=1$ and $A=2$) along which the two mouths are to move. Constrain each of these world lines to satisfy $g_A \ll 1/b$, where g_A is the magnitude of its 4-acceleration and b is the radius of the wormhole mouths. Mark off along world line A proper time τ as measured by an ideal clock in the external, Minkowski spacetime $[d\tau^2=dT^2-dX^2-dY^2-dZ^2$ where (T,X,Y,Z) are Lorentz coordinates of the external spacetime]; and choose the origin of τ in some arbitrary manner. Through each point τ on world line A there passes a unique flat, 3-dimensional spacelike hypersurface $\mathcal{S}_A(\tau)$ that is orthogonal to the world line—the "slice of simultaneity" as seen by the wormhole mouth at time τ. On this $\mathcal{S}_A(\tau)$ cut out a ball with radius b, centered on the world line, and denote by $\mathcal{B}_A(\tau)$ the surface of that ball. Then identify the surface $\mathcal{B}_1(\tau)$ with the surface $\mathcal{B}_2(\tau)$ in such a way that (i) (aside from unavoidable errors that go to zero as $g_1 b \to 0$ and $g_2 b \to 0$) the intrinsic 3-geometries of the world tubes

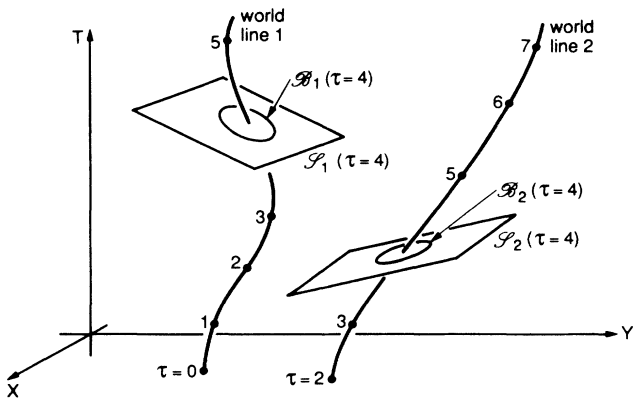


FIG. 2. Method for constructing a simple model of a wormhole with mouths that move along arbitrary, but weakly accelerated, world lines in flat spacetime.

$\mathcal{B}_1 \equiv \{\mathcal{B}_1(\tau), \quad -\infty<\tau<+\infty\}$ and $\mathcal{B}_2 \equiv \{\mathcal{B}_2(\tau), -\infty<\tau<+\infty\}$ agree; and (ii) parity is preserved for particles that go through $\mathcal{B}_1=\mathcal{B}_2\equiv\mathcal{B}$. The result is an orientable wormhole with mouths $\mathcal{B}_1(\tau)$ and $\mathcal{B}_2(\tau)$ connected by an infinitesimally thin throat.

This wormhole spacetime has the attractive feature that it is everywhere flat, except on the wormhole's world tube $\mathcal{B}$. On that tube there is a discontinuity of the extrinsic curvature

$$[K_{ac}] = -\frac{2}{b}\gamma_{ac} , \tag{1}$$

and a corresponding delta-function-localized Riemann curvature of spacetime

$$R_{alcl} = -\frac{2}{b}\gamma_{ac}\delta(l) . \tag{2}$$

Here γ_{ac} is the intrinsic metric of the 2-spheres of $\mathcal{B}(\tau)$, l is proper radial distance through $\mathcal{B}$, and Latin indices from the early part of the alphabet denote components in $\mathcal{B}(\tau)$ while the l index denotes a unit radial component. [In Eqs. (1) and (2) there are errors caused by the accelerations of the wormhole's mouths. When the wormhole's throat is smoothed so it has a finite length $\Delta l \sim b\sqrt{gb}$ where g is the maximum of g_1 and g_2, then for $gb \to 0$ those errors become vanishingly small compared to the terms included in (1) and (2).]

Because spacetime is everywhere flat except on $\mathcal{B}$, it is especially easy to solve $\Box\Phi=0$ on this spacetime: One can solve it using a flat-space propagator (or separation of variables or other flat-spacetime technique), together with appropriate junction conditions across $\mathcal{B}$. The junction conditions, obtained by integrating $\Box\Phi=0$ over the interior of a thin "pillbox" centered on $\mathcal{B}$, are

$$\Phi \text{ and } \Phi_{,l} \text{ both continuous across } \mathcal{B} . \tag{3}$$

Note that l increases from $-\infty$ to zero as one moves along $\mathcal{S}_1(\tau)$ from spacelike infinity to mouth 1, $\mathcal{B}_1(\tau)$; and it then increases onward from zero to $+\infty$ as one moves from mouth 2, $\mathcal{B}_2(\tau)$, on outward to spacelike infinity; cf. Fig. 1 (where, however, the throat has finite rather than infinitesimal length). In practical calculations one may prefer to work with Euclidean radial coordinates r_1 and r_2 which increase from b to ∞ as one moves radially outward from the two mouths, In this case the boundary conditions (3) become

$$\Phi \text{ continuous}, \quad \frac{\partial\Phi}{\partial r_1} = -\frac{\partial\Phi}{\partial r_2} . \tag{3'}$$

From these junction conditions one can readily work out the reflection and transmission coefficients for waves which impinge on one of the mouths. For example, in the case of spherical waves (angular momentum quantum number 0) with angular frequency ω which impinge on mouth 1, the waves ϕ_1 near mouth 1 and ϕ_2 near 2 are given by

$$\phi_1 = \left[\frac{e^{-i\omega r_1}}{r_1} + R\frac{e^{+i\omega r_1}}{r_1} \right] e^{-i\omega\tau} , \tag{4a}$$

$$\phi_2 = T\frac{e^{i\omega r_2}}{r_2} e^{-i\omega\tau} , \tag{4b}$$

where

$$T = \frac{-i\omega b}{1-i\omega b} e^{-2i\omega b} , \quad R = -\frac{1}{1-i\omega b} e^{-2i\omega b} , \tag{4c}$$

and τ has been carried into the 4-dimensional spacetime region near each mouth using the hypersurfaces $\mathcal{S}_A(\tau)$. (Here, as throughout, there are tiny errors that go to zero as the mouths' accelerations g_A become vanishingly small compared to $1/b$.[18]) The transmission and reflection coefficients (4c) have the usual property that $|T|$ becomes unity and $|R|$ zero in the geometric optics limit, $\omega b \gg 1$; and $|T|$ becomes zero and $|R|$ unity in the opposite limit of waves of very low frequency, $\omega b \ll 1$.

When dealing with wavepackets whose transverse sizes are small compared to b (geometric-optics limit), the boundary conditions (3') imply that the wormhole acts precisely like a diverging lens with focal length $f = b/2$.

B. Specific examples of wormhole spacetimes

Figure 3 shows three specific examples of wormhole spacetimes constructed in the above manner. In all three examples the closest the mouths ever come to each other (distance a as measured in the center-of-mass frame) is large compared to the wormhole radius b:

$$a \gg b . \tag{5}$$

Correspondingly, the wormhole radii are so small in Fig. 3 that one cannot see them; all one sees is the world lines along which the mouths move.

The example in Fig. 3(a) was studied in detail in Ref. 1. The two mouths are initially at rest in the external Lorentz frame (T,X,Y,Z) with separation $a \gg b$ and with proper times $\tau = T$ at both mouths. Then mouth 1 remains inertial while mouth 2 undergoes a "twins-paradox" trip of high-speed outward motion and return (but with small acceleration). During the return trip CTC's form; there are no CTC's to the past of the Cauchy horizon $\mathcal{H}$, and there exist CTC's through every point in the future of $\mathcal{H}$. The horizon $\mathcal{H}$ is generated by future-directed null geodesics which peel off of a unique, closed null geodesic $\mathcal{C}$ and then, after many transits through the wormhole, escape into the external spacetime from mouth 1; see the Appendix for further detail. As was discussed in Ref. 1, the diverging-lens behavior of the wormhole protects this Cauchy horizon against the type of classical instability (classical waves getting amplified indefinitely as they near it) that characterizes all Cauchy horizons encountered previously in general relativity. However (as we have only realized since Ref. 1 was written), for this specific example, the wormhole's diverging-lens behavior seems not to protect the Cauchy horizon against the analogous quantum instability: there seems to be a divergent amplification of vacuum fluctuations as they circulate through the wormhole many times just before reaching the Cauchy horizon's closed null geodesic $\mathcal{C}$, and a corresponding divergent vacuum polarization at $\mathcal{C}$.[4] [See Ref. 19 for this effect in a two-dimensional spacetime analogous to Fig. 3(a).] Whether such quantum instabilities occur generically for the Cauchy horizons of wormhole spacetimes, and whether they are all encompassing enough and strong enough to prevent the formation of CTC's is not yet known. In this paper we ignore this possibility.

In Fig. 3(b) mouth 1 is always at rest at the origin of

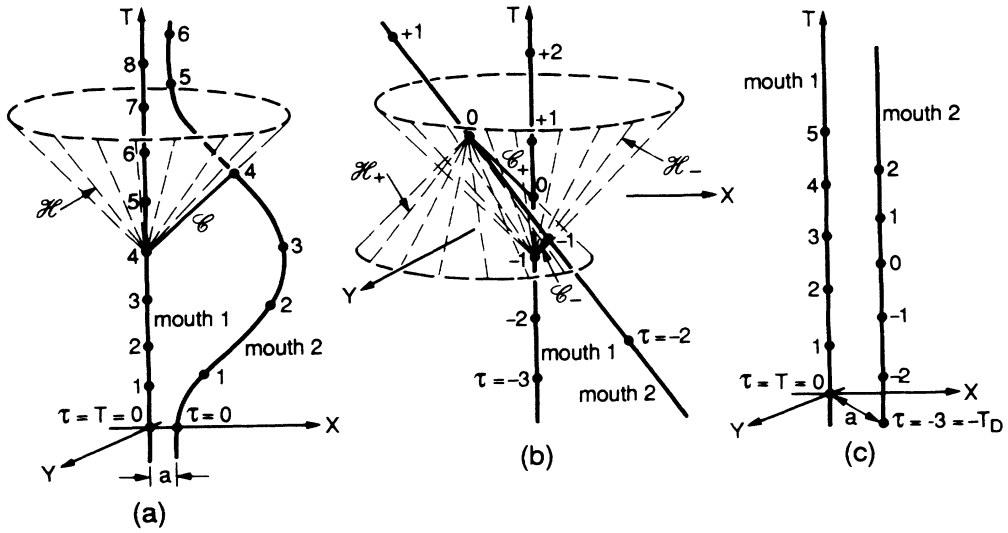


FIG. 3. Three examples of wormhole spacetimes constructed in the manner of Fig. 2. (a) The "twins-paradox" example studied in Ref. 1. The CTC's occur only to the future of the Cauchy horizon $\mathcal{H}$. (b) Both mouths move inertially, with uniform relative velocity. In this case the CTC's are confined to the region between the two horizons $\mathcal{H}_-$ and $\mathcal{H}_+$. (c) The mouths are at rest with respect to each other, and there are CTC's throughout the spacetime.

the (T,X,Y,Z) coordinate system with $\tau=T$, while mouth 2 moves always with uniform velocity $-v\mathbf{e}_X$ along the line $(Y=a, Z=0)$, passing closest to mouth 1 at time $T=0$. If, at the point of closest approach, proper time at mouth 2 reads $|\tau|<a\sqrt{2/(\gamma+1)}$, then this spacetime never possesses any CTC's. But if mouth 2 reads $|\tau|>a\sqrt{2/(\gamma+1)}$ at closest approach, then there will be a bounded region with CTC's. Figure 3(b) shows the case where mouth 2 reads $\tau<-a\sqrt{2/(\gamma+1)}<0$ at closest approach. In this case the region with CTC's is bounded between an initial Cauchy horizon $\mathcal{H}_-$ and a final Cauchy horizon $\mathcal{H}_+$. If the speed of mouth 2 is not too close to the speed of light, and perhaps also for arbitrarily high speeds (see Appendix), the initial horizon $\mathcal{H}_-$ is generated by future-directed null geodesics that peel off a single closed null curve $\mathcal{C}_-$ and, after many wormhole transits, escape into the external spacetime from mouth 1. Similarly, the final horizon $\mathcal{H}_+$ is generated by past-directed null geodesics that peel off the closed null curve $\mathcal{C}_+$ and then escape into the external spacetime from mouth 2.

In Fig. 3(c) both mouths are forever at rest in the external Lorentz frame (T,X,Y,Z), with their crossings of the hypersurface $T=0$ at $\tau=0$ for mouth 1 and at $\tau=-T_d$ (the "delay time") for mouth 2. In this case, for $|T_d|>a$ (the case shown in the figure) the entire spacetime possesses CTC's, while for $|T_d|\le a$ there are no CTC's.

Other interesting examples of wormholes with CTC's in flat external spacetimes are discussed in Ref. 2; and examples in curved spacetimes are discussed in Ref. 3.

C. The Cauchy problem with initial data at past null infinity

Turn attention, now, to the Cauchy problem for $\Box\Phi=0$ in these spacetimes. We shall discuss the Cauchy problem successively for initial data posed at past null infinity of Figs. 3(a)–3(c) (this section), on a global spacelike hypersurface before the Cauchy horizons of Figs. 3(a) and 3(b) (Sec. II D), and on hypersurfaces after the Cauchy horizons of Figs. 3(a) and 3(b) and throughout Fig. 3(c) (Sec. II E).

At first one might worry that the data at $\mathcal{I}^-$ are not sufficient to determine the full evolution of Φ. This is because the characteristics along which Φ propagates are future-directed null geodesics, and not all such geodesics originate at $\mathcal{I}^-$. For example, for case (a) of Fig. 3 the future-directed null geodesics that generate the Cauchy horizon $\mathcal{H}$ all enter the spacetime from $\mathcal{C}$ rather than from $\mathcal{I}^-$. However, as was discussed in Ref. 1, no new field Φ can enter the spacetime from $\mathcal{C}$ along with these null geodesics. From one point of view this is because these geodesics are a set of measure zero. From another, more rigorous point of view it is because of the diverging-lens property of the wormhole: These geodesics (which are past incomplete), and any field that enters the spacetime with them, loop through the wormhole an infinite number of times as they peel off $\mathcal{C}$. With each loop the amplitude of the field Φ goes down by a factor $b/2a$, and its frequency goes up by $\sqrt{(1+v)/(1-v)}$, where v is the speed of mouth 2 as it passes through $\mathcal{H}$. We insist that $(b/2a)\sqrt{(1+v)/(1-v)}$ be small compared to unity. This guarantees (i) that any field which tries to enter the spacetime from $\mathcal{C}$ will be driven to zero amplitude and zero energy before it can get into the spacetime, and (ii) that any (classical) field which originates at $\mathcal{I}^-$ will evolve through the Cauchy horizon with finite amplitude and finite stress-energy tensor, and therefore will not destabilize the Cauchy horizon.[1]

There are also future-directed null geodesics that enter the spacetime at future timelike infinity i^+ and, propagating always toward the local future, work their way down to the vicinity of $\mathcal{H}$ via an infinite number of wormhole traversals. However, again because of the wormhole's diverging-lens property, no new field Φ can enter the spacetime by this route.

We note in passing that by inserting a waveguide between the wormhole mouths and keeping it there forever into the future, one would be able to counteract the wormhole's diverging-lens behavior and thereby bring new field in from future timelike infinity i^+. In this case a full set of initial data would have to include those coming from i^+ as well as those from $\mathcal{I}^-$. We shall not discuss this possibility further.

If there were no CTC's, then the full spacetime development of Φ would be uniquely determined by giving the value of the field $r\Phi$ (where $r=\sqrt{X^2+Y^2+Z^2}$) everywhere on $\mathcal{I}^-$. Do the existence of CTC's and the principle of self-consistency constrain these initial data in any way? Friedman and Morris[11] have proved that the answer is *no* for the simplest of our spacetimes: that of Fig. 3(c). We thus are sure in this case, and we are almost certain also for Figs. 3(a) and 3(b), that even though there are CTC's, the full development of Φ is everywhere uniquely determined by the standard initial data on $\mathcal{I}^-$.

To see why this should be so for all three spacetimes in Fig. 3, imagine solving $\Box\Phi=0$ by straightforward local evolution (e.g., with Huygens' principle, or by numerical integrations using an arbitrarily fine mesh). The field will propagate as in any flat spacetime until it encounters a wormhole mouth. At the mouth the field is subjected to the junction conditions (3), which reflect some of the field and transmit some. It is not important whether, from the viewpoint of external time T, the transmitted field emerges from the wormhole before it entered or afterward. In either case the transmitted field just continues to propagate, superposing linearly on field that has followed other routes.

To help keep track of this conceptually, denote by $\Phi^{(0)}$ that field which has never scattered from either wormhole mouth; by $\Phi_1^{(1)}$ that which has scattered just once and is outgoing from mouth 1 (it is comprised of a reflection of $\Phi^{(0)}$ from off of the wormhole at mouth 1 and also a transmission of $\Phi^{(0)}$ that entered the wormhole at mouth 2); by $\Phi_2^{(1)}$ that which has scattered just once from the wormhole and is outgoing from mouth 2 (it is comprised of a reflection of $\Phi^{(0)}$ from off of the wormhole at mouth 2 and also a transmission of $\Phi^{(0)}$ that entered the wormhole at mouth 1). That is, each component $\Phi_i^{(k)}$ originates at mouth i where it is fed in by a reflection of $\Phi_j^{(k-1)}$ and by a transmission of $\Phi_i^{(k-1)}$ (here $i,j=1,2$ and $i\neq j$) via the boundary conditions (3), then it propa-

gates via the flat-space wave equation with some losses down mouth j (feeding $\Phi_i^{(k+1)}$ and $\Phi_j^{(k+1)}$) and other losses to future null infinity. At any event $\mathcal{P}$ in spacetime the full field Φ is the sum of all the components $\Phi_i^{(k)}$ there, given by $\Phi=\Phi^{(0)}+\sum_{k=1}^{\infty}(\Phi_1^{(k)}+\Phi_2^{(k)})$. If this sum converges for all possible choices of the initial data at $\mathcal{I}^-$, then we can be quite confident that self-consistency leaves the initial data unconstrained.

With each scattering from the wormhole and propagation on out to a typical point in spacetime, the amplitude of the high-frequency ($\omega \gtrsim 1/b$) parts will go down by a factor of order $b/2a$ because of the wormhole's diverging-lens property; and the low-frequency parts ($\omega \ll 1/b$) will attenuate so much [cf. Eqs. (4c)] that their scattering and propagation on out to our typical point will reduce their amplitude even more strongly than by $b/2a$. Therefore, it seems reasonable to expect that the sum over $\Phi_i^{(k)}$ will be a convergent power series in the small parameter $b/2a$; and correspondingly, it seems fairly clear that *the principle of self-consistency places no constraints on initial data at past null infinity, $\mathcal{I}^-$*. For every choice of initial data on $\mathcal{I}^-$ there will exist a unique, global solution to $\Box\Phi=0$. (That solution typically will not be smooth at the Cauchy horizon $\mathcal{H}$: Just before $\mathcal{H}$, a high-frequency field can circulate through the wormhole indefinitely many times on null geodesics that asymptotically approach the closed null curve $\mathcal{C}$. With each circuit the field's amplitude goes down by a factor $b/2a$ and its frequency goes up by $\sqrt{(1+v)/(1-v)}$. Correspondingly, $\partial^n\Phi/\partial t^n$ will be finite at $\mathcal{H}$ for all

$$n < \frac{-\ln(b/2a)}{\ln\sqrt{(1+v)/(1-v)}} , \tag{6}$$

but it may be infinite for n larger than this critical value.)

The Friedman-Morris proof,[11] for case (c) of Fig. 3, that the principle of self-consistency does not constrain the initial data on $\mathcal{I}^-$ in any way, begins by showing the existence of generalized eigenfunctions of the operator $\nabla^2+\omega^2$ in the space of functions that are smooth outside the wormhole throat $\mathcal{B}$, obey the boundary conditions (3), and fall off at large r like $1/r$. Then the self-consistent evolution for Φ is obtained as a convergent superposition of the generalized eigenfunctions. The proof shows, moreover, that not only does there exist a self-consistent evolution of Φ throughout the spacetime for each asymptotically regular choice of initial data at $\mathcal{I}^-$; but the evolution in fact is unique. Although the proof does not rely on the iteration procedure outlined above, Friedman and Morris also show convergence of the iteration in the long-wavelength regime [for $\lambda > \sqrt{2}e\pi b$ and $b/a < 1/(2e)$] and in the short-wavelength limit. For intermediate wavelengths, they have verified numerically that the iteration converges, but a formal proof is not yet in hand. We fully expect the Cauchy problem for the scalar-wave equation in cases (a) and (b) to be similarly well defined.

More generally, we expect that for other classical zero-rest-mass fields (e.g., the electromagnetic field) in the spacetimes (a)–(c), the standard type of initial data at $\mathcal{I}^-$ will produce self-consistent evolutions that are unique up to the addition of fields with topological charge. For example, to achieve uniqueness for the electromagnetic field, one must specify two numbers, the total electric and magnetic fluxes through the wormhole's throat, in addition to the initial, radiative data at $\mathcal{I}^-$.

To make this more clear, we note that for spacetimes that are asymptotically static in the past, topological charges are associated with solutions that are stationary in the past and thus have nonzero data on past timelike infinity i^-. Scalar fields, by contrast with electromagnetic fields, do not have topological charges for wormhole spacetimes, and it is easy to see that they have no asymptotically static solutions on static spacetimes: The scalar wave equation takes the form of a 3-dimensional, negative definite Laplacian

$$\nabla^2\Phi=0 \Longrightarrow \int_{\mathcal{V}} \Phi\nabla^2\Phi = -\int_{\mathcal{V}} (\nabla\Phi)^2 = 0 , \tag{7}$$

so $\Phi=0$ at spatial infinity means that Φ must vanish everywhere.

Note that in some asymptotically flat spacetimes, even ones with Euclidean topology, data on $\mathcal{I}^-$ can fail to be complete.[20] One example is a spacetime with an ergosphere and no horizon. Counter-rotating particles in the ergosphere have negative energy and are therefore trapped in the past and the future. Corresponding initial data for fields with negative energy always exist, describing counter-rotating waves in the ergosphere. When evolved to the past, such negative-energy fields must have support at i^- as well as at $\mathcal{I}^-$, because data at $\mathcal{I}^-$ can have only positive energy. In fact, such spacetimes have unstable outgoing normal modes that vanish exponentially as one approaches $\mathcal{I}^-$ and i^-, and thus even data on $\mathcal{I}^- \cup i^-$ fail to be complete. We do not expect any such peculiarities in our wormhole spacetimes; rather, because geodesics that are spatially trapped are a set of measure zero, we expect that data on $\mathcal{I}^-$ will be complete in the sense of the preceding several paragraphs.

D. Initial data posed on a global spacelike hypersurface preceding the Cauchy horizon

In cases (a) and (b), where there is a Cauchy horizon before which CTC's do not exist, we can try to pose initial data in the region preceding the horizon. In this region there exist global spacelike hypersurfaces which pass smoothly through the wormhole and intersect spacelike infinity i_0 in 2-surfaces with 2-sphere topology. Such a hypersurface, Σ, is a natural place on which to pose the initial data. If there were no CTC's to the future of Σ, then a complete set of initial data would be the values of Φ and its normal derivative $\Phi_{,n}$ on Σ; and to each choice of $\{\Phi, \Phi_{,n}\}$ there would be a unique future evolution of Φ.

Can the CTC's in Figs. 3(a) and 3(b) change this? The answer is almost certainly *no*. Again, as in the last section, we can evolve our arbitrary initial data forward as a sum of components that have traversed the wormhole in specific sequences of ways; and as in the last section, it is reasonable to expect the sum to be a convergent power series in $b/2a$.[11]

E. Initial data posed in the region with CTC's

Denote by $\mathcal{T}$ the region of spacetime with CTC's—i.e., the future of $\mathcal{H}$ in case (a); the region between $\mathcal{H}_-$ and $\mathcal{H}_+$ in case (b); the entire spacetime in case (c). The posing of initial data in this region $\mathcal{T}$ is made complicated by the complex structure of global spacelike hypersurfaces there: Any spacelike hypersurface that enters the region $\mathcal{T}$ must inevitably encounter and pass into mouth 1, emerging then from mouth 2 at a location which is in the causal future of its entry; it must then again encounter mouth 1, and again emerge from mouth 2 at a location still further in the future, and so forth. As a result the spacelike hypersurface will have many [and in cases (a) and (c) an infinite number of] sheets, each extending out to a 2-surface with 2-sphere topology at spacelike infinity. Such a hypersurface is ill suited for posing initial data for the Cauchy problem.

It seems to us that the difficulties of the Cauchy problem for Φ in the region $\mathcal{T}$ are best regarded as arising from this failure of $\mathcal{T}$ to have reasonable spacelike hypersurfaces, and not from the CTC's themselves. If the CTC's were the key impediment, then the Cauchy problem with data posed before $\mathcal{T}$ would also encounter difficulties, which it seems not to (Secs. II C and II D above).

It should be evident that the difficulties with posing data in $\mathcal{T}$ are global and not local. The difficulties are sufficiently nonlocal for it to seem clear intuitively that the spacetimes of Fig. 3 are benign for the field Φ in the sense discussed in Sec. I. We give a proof of benignness, in the geometric optics limit, at the end of this section; but first we shall lay foundations for that proof.

We have been able fully to understand the Cauchy problem in the region $\mathcal{T}$ only in the geometric optics limit of arbitrarily short wavelengths (propagation of Φ along null rays). We shall describe it in this limit, restricting attention for pedagogical simplicity to case (c) of Fig. 3—which we redraw with auxiliary information in Fig. 4.

If the wormholes were absent, we could pose our initial data, Φ and $\partial\Phi/\partial T$, on the hypersurface Σ of constant external time $T=0$; and those data would propagate forward in time along null rays to produce a unique field Φ throughout the future of Σ. In the presence of the wormhole we shall similarly choose as our initial data Φ and $\partial\Phi/\partial T$ on Σ; and then we shall adjust those data and provide additional data elsewhere so as to preserve existence and uniqueness of the evolution.

The data on Σ are not sufficient to determine the evolution uniquely because there are many rays which never pass through Σ. An example is the ray α in Fig. 4. Fields can propagate along such rays, transiting from the past of Σ to its future via the wormhole throat rather than via Σ itself. Correspondingly, we must augment our initial data by the field Φ on such rays as they pass through the throat. Stated more precisely, we must give, on the portion $\mathcal{B}_* \equiv \mathcal{B}(-T_d < \tau < 0)$ of the wormhole's world tube, the field Φ that is propagating from mouth 1 to mouth 2. Notice that the combined surface $\mathcal{S} \equiv \Sigma \cup \mathcal{B}_*$ on which we give our full data (and which is stippled in Fig. 4) is a continuous surface that has as its only boundary a 2-sphere at spacelike infinity i_0. It is the closest thing there is to a Cauchy surface in the region $\mathcal{T}$ of CTC's: it extends across the entire spacetime, but it is not everywhere spacelike; rather, it contains a spacelike piece Σ and a timelike piece $\mathcal{B}_*$.

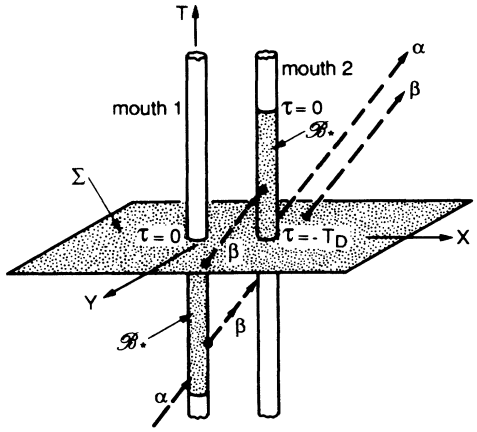


FIG. 4. The Cauchy problem for a classical scalar field Φ in the geometric optics limit, with the initial data posed in the region $\mathcal{T}$ of CTC's. The spacetime is that of Fig. 3(c). The initial data are posed on the stippled region (the spacelike hypersurface Σ cut off at the wormhole throat, i.e., at the 2-spheres labeled $\tau=0$ and $\tau=-T_D$, and the segment $\mathcal{B}_*$ of the throat's timelike world tube, which joins these two spheres). The data are not fully free; rather, they are constrained so that field which propagates along rays such as β will be fully self-consistent.

It is easy to convince oneself that the data we have posed on the surface $\mathcal{S}$ are sufficient to predict the future. However, not all such initial data will produce self-consistent evolution. The reason is that there are rays such as β (Fig. 4) which, by traversing the wormhole from mouth 2 to mouth 1 (opposite direction from ray α), manage to pass through Σ more than once. The initial data on such rays must be constrained so as to give self-consistent evolution; and similarly for rays (if any) that pass through $\mathcal{B}_*$ from mouth 1 to mouth 2 and then, via multiple wormhole transits in the opposite direction, manage to get into the past of Σ, and then cross the stippled region a second time.

To recapitulate, the initial data must be posed on the stippled region $\mathcal{S} = \Sigma \cup \mathcal{B}_*$ of Fig 4, with Φ and $\partial\Phi/\partial T$ given on Σ and mouth-1-to-mouth-2 field Φ given on $\mathcal{B}_*$; and those portions of the initial data which propagate along rays that intersect the stippled region two or more times must be constrained for self-consistency.

It can be verified as follows that these self-consistency constraints are benign. Consider, in the spacetime of Fig. 4 (and also in the spacetimes of Fig. 3) the set $\mathcal{G}$ of all events $\mathcal{Q}$ with the property that some null geodesic through $\mathcal{Q}$, when followed forward or backward in local time, ultimately returns arbitrarily close to $\mathcal{Q}$. Examples of such events are those which lie on the closed null geodesic $\mathcal{C}$ in Fig. 3(a); in other words, $\mathcal{C} \subset \mathcal{G}$. It should be clear that on any spacelike hypersurface through the

spacetimes of Figs. 3 and 4, the set $\mathcal{G}$ has measure zero. Now, choose an arbitrary event $\mathcal{P}$ which is not in the set $\mathcal{G}$, and choose an arbitrary spacelike hypersurface $\mathcal{S}$ through that event. Then there will be some neighborhood $\mathcal{N}$ of $\mathcal{P}$ in $\mathcal{S}$, which (i) can be made part of a (slightly locally deformed) hypersurface that has the same form as the Σ of Fig. 4; and (ii) has the property that no null geodesic intersects $\mathcal{N}$ more than once. The initial data for evolution of Φ can be posed arbitrarily on this neighborhood $\mathcal{N}$ (see preceding paragraph). Those initial data determine, through each event in $\mathcal{N}$, a future directed null ray along which the field from that event evolves via geometric optics. By evolving Φ along these null rays, and adding together (linearly superposing) the resulting fields at all events in spacetime through which more than one of these rays pass, and by setting to zero the field Φ at events through which none of these rays pass, we obtain a self-consistent evolution which contains our initial data on $\mathcal{N}$. Thus, when we ignore the set $\mathcal{G}$, the spacetime is benign for the field Φ. But, since $\mathcal{G}$ has measure zero, when we insist that our field be made of nonzero wavelengths (as any field must), but still contain only tiny wavelengths (so geometric optics remains a good approximation), then data posed in neighborhoods of all events not in $\mathcal{G}$ will completely determine the evolution of Φ. Thus, the events in $\mathcal{G}$ cannot prevent the spacetime from being benign. (See also the second paragraph of Sec. II C, above, for a different viewpoint on the unimportance of events like those in $\mathcal{G}$.)

In this proof of benignness, it is crucial that the set $\mathcal{G}$ have measure zero. If it did not have measure zero, then not only would the spacetime fail to be benign, but also finite-wavelength waves presumably could propagate around and around the spacetime and pile up an infinite number of times in the vicinities of events in $\mathcal{G}$. This presumably would produce a divergence of Φ's stress-energy tensor, which would act back through the Einstein field equations to change the structure of the spacetime. This leads us to suspect that classical stability of the evolution of Φ may be a key underpinning for benignness; see the discussion and conjecture in Sec. I.

F. Global conservation laws in wormhole spacetimes

Turn attention from the Cauchy problem for the field Φ to its law of energy conservation. In any spacetime, including one with wormholes and CTC's, the stress-energy tensor of Φ is given by

$$T_{\mu\nu}=\frac{1}{2}(2\Phi_{;\mu}\Phi_{;\nu}-g_{\mu\nu}\Phi_{;\alpha}\Phi^{;\alpha}) . \tag{8}$$

Here the semicolons denote covariant derivatives. The field equation $\Box\Phi \equiv \Phi^{;\alpha}{}_{;\alpha}=0$ enforces the local law of energy-momentum conservation $T^{\mu\nu}{}_{;\nu}=0$. If the spacetime has a timelike Killing vector field ξ^{μ}, there will be a corresponding conserved energy E with density-flux 4-vector $S^{\mu}=-T^{\mu\nu}\xi_{\nu}$; i.e., this S^{μ} will satisfy

$$S^{\mu}{}_{;\mu}=0 . \tag{9a}$$

From this differential conservation law, by integrating over any closed 4-volume $\mathcal{V}$ and applying Gauss's law, one can obtain an integral conservation law for the energy:

$$\int_{\partial\mathcal{V}} S^{\mu}d\Sigma_{\mu}=0 . \tag{9b}$$

Here $\partial\mathcal{V}$ is the boundary of $\mathcal{V}$.

Although Gauss's law is commonly stated only for orientable manifolds, when S^{μ} is a true vector field (i.e., polar, not axial), Eq. (9b) follows from (9a) for nonorientable manifolds as well,[21] with $d\Sigma_{\mu}$ the outward directed volume element normal to $\partial\mathcal{V}$.

The wormhole spacetime of Fig. 3(c), which has CTC's throughout, possesses a timelike Killing vector field: $\xi=\partial/\partial T$, where T is the Lorentz time coordinate. (The discontinuity in T across the wormhole throat does not produce any corresponding discontinuity in $\xi=\partial/\partial T$; that ξ satisfies Killing's equation everywhere, including at the throat.) The corresponding density-flux 4-vector $S^{\mu}=-T^{\alpha\mu}\xi_{\alpha}$ has Lorentz components

$$S^{\mu}=T^{T\mu} ; \tag{10}$$

i.e., the density of conserved energy is just the ordinary energy density T^{TT} measured by static observers, and the flux of conserved energy is the ordinary energy flux T^{Tj} that they measure.

Now, apply the integral conservation law to the 4-volume $\mathcal{V}$ depicted in Fig. 5. The boundary of $\mathcal{V}$ includes two copies of the stippled region $\mathcal{S}\equiv\Sigma\cup\mathcal{B}_*$ on which we posed initial data in the last section: one copy $\mathcal{S}_1$ with Σ_1 the surface $T=T_1$, and the other copy $\mathcal{S}_2$ with Σ_2 the surface $T=T_2>T_1$. These $\mathcal{S}_1$ and $\mathcal{S}_2$ are attached together by a timelike spherical world tube at spacelike infinity i^0 to form the closed 3-surface $\partial\mathcal{V}$. When this $\partial\mathcal{V}$ is used in the energy-conservation law (9b), and the outward normal is reversed on $\mathcal{S}_2$, and it is presumed that $T^{\mu\nu}$ vanishes at spacelike infinity, then we obtain

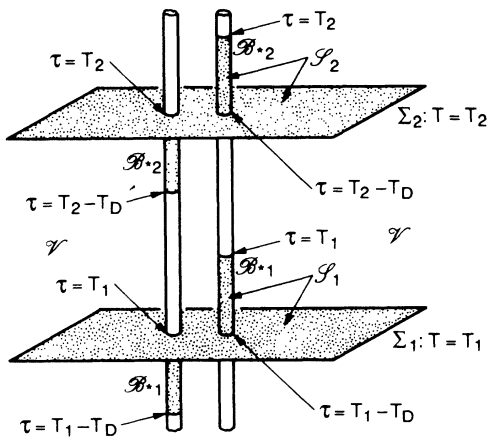


FIG. 5. Region of integration for formulating the law of global conservation of energy in a static, asymptotically flat spacetime with a wormhole and closed timelike curves. Each surface $\mathcal{S}$ is identical in geometry to the stippled region in Fig. 4, where initial data are posed. The integral of the energy density-flux 4-vector over each surface $\mathcal{S}$ has a value independent of $\mathcal{S}$, i.e., a conserved value; that value is the total energy in the spacetime.

$$\int_{\mathcal{S}_2} S^\mu d\Sigma_\mu = \int_{\mathcal{S}_1} S^\mu d\Sigma_\mu \ . \tag{11}$$

In other words, the quantity

$$E = \int_{\mathcal{S}} S^\mu d\Sigma_\mu = \int_\Sigma T^{TT} dx\, dy\, dz + \int_{\mathcal{B}_*} T^{Tr} dA\, dT \tag{12}$$

is independent of the time T at which the surface Σ is taken. (Here T^{Tr} is the radial energy flux flowing through the wormhole throat from mouth 1 to mouth 2, and dA is the element of surface area on the throat $\mathcal{B}_*$.)

This conserved quantity, E, is the field's total energy, and Eq. (12) for it has a simple interpretation: The integral over Σ is the total energy measured by static observers at time T. The integral over $\mathcal{B}_*$ consists of two parts: a positive contribution due to energy which flows from mouth 1 to mouth 2 ("time travel into the future") thereby avoiding the surface Σ; and a negative contribution due to energy which flows from mouth 2 to mouth 1 ("time travel into the past") thereby forcing itself to cross the surface Σ twice. Equation (12) says that the conserved total energy is the energy seen by static observers at time T (i.e., on Σ), plus the energy which is absent at time T because it is time traveling into the future to avoid showing up there, minus the energy that is double counted at time T because it time traveled into the past and thence flowed through Σ twice.

The same procedure as we have applied to the conserved energy density-flux 4-vector S^μ can be applied to any other conserved vector field. In other words, just as in the absence of wormholes and CTC's, so also in their presence, every differential conservation law that involves a density-flux 4-vector (e.g., charge conservation and baryon conservation) can be converted into an integral conservation law. As we remarked in Sec. I, this fact reinforces the authors' feelings that CTC's are not so nasty as people generally have assumed.

G. Nonlinear effects: The billiard ball problems

To what extent is the classical, massless scalar field Φ a good guide to the Cauchy problem for other fields? There are two important properties that Φ lacks: Interaction with itself or other fields, and quantum-mechanical behavior. Elsewhere one of the authors (Novikov[22]) discusses a class of complicated classical model problems with interactions (e.g., bombs that explode in response to a trigger signal, and then send explosive debris through a wormhole and backward in time where it tries to trigger the explosion before the explosion actually occurred). In this and the next section we shall pose and discuss a class of much simpler model problems by which interaction effects and first-quantization effects can be studied (but not second-quantization effects, i.e., not quantum field theory).

Our model problems involve a single, perfectly elastic "billiard ball" (particle surrounded by a hard-sphere, two-body potential), which moves relative to the wormhole mouths with speeds small compared to light so it can be treated nonrelativistically. By traveling backwards in time through a wormhole, the ball can encounter and collide with itself. We shall discuss such a ball classically at first, and then quantum mechanically.

Figure 6(a) shows a "paradox," pointed out to us by Polchinski,[23] which started us thinking about billiard-ball problems: The spacetime is that of Fig. 3(a), and Fig. 6(a) is a spatial diagram in the final wormhole rest frame, showing events that occur long after mouth 2 has come to rest. The initial data (initial position and velocity of the ball) are posed before the Cauchy horizon in such a way that the ball moves along the solid trajectory α. This trajectory takes the ball into mouth 2 at point A, then out of mouth 1 at B before it went in, and then along the dashed trajectory β. The timing is just right for the ball to hit itself, knocking itself along the dotted trajectory γ and thereby preventing itself from ever reaching mouth 2.

This paradox is an idealized version of "killing one's younger self" (changing the past). The principle of self-consistency says that such evolution is impossible. The history shown in Fig. 6(a) is not a self-consistent solution to the evolution equations; in fact, strictly speaking it is not a solution at all because the world line of the billiard ball is double valued in a self-inconsistent way. If there are no globally self-consistent trajectories that start with the initial world line α, then the principle of self-consistency will prohibit the initial trajectory α from being posed in the first place. If there are global solutions, then the initial trajectory will be permitted.

It is a well-posed (but somewhat algebraically complicated) problem to ask whether there exist any self-consistent evolutions that begin with the initial trajectory α—and if so, whether there is only one (uniqueness) or more than one. Two of the authors (Echeverria and Klinkhammer[13]) are currently studying this question, and have demonstrated that for a wide class of initial data which give paradoxical, self-inconsistent solutions of the form shown in Fig. 6(a), there in addition are self-consistent solutions with the qualitative form shown in Fig. 6(b): The ball starts out on the trajectory α; but before reaching mouth 1 it is hit by itself moving along a trajectory β' which is a bit different than the β of Fig. 6(a). The ball on β' strikes itself on α a gentle, glancing blow, driving itself into a slightly altered trajectory α'. This altered trajectory takes the ball down mouth 1 at a slightly altered point A', and out at B' before it went

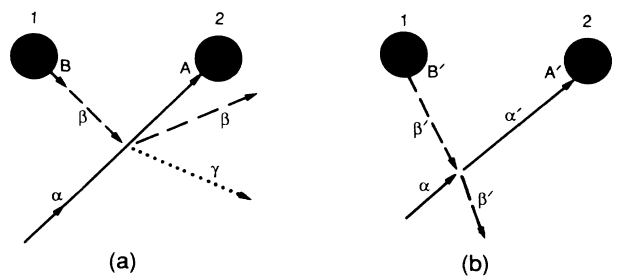


FIG. 6. Spatial trajectories of a billiard ball that travels backward in external time by traversing a wormhole, and then collides with itself. The evolution depicted in (a) violates the principle of self-consistency; that in (b) does not.

down, and along the trajectory β' to the event of collision.

Echeverria and Klinkhammer[13] have reduced the search for self-consistent evolutions, for given initial data (initial billiard-ball trajectory), to the solution of a highly nonlinear set of algebraic equations. There is the same number of unknowns as equations, so there should be a discrete number of solutions (evolutions): 0,1,2,. . . . For most initial data, Echeverria and Klinkhammer[13] have found one or more solutions; but there may well be some initial data with zero solutions, i.e., no self-consistent evolutions. An analogous possible absence of self-consistent evolutions is being analyzed by Novikov and Petrova[24] for an inelastic billiard ball with friction.

For a perfectly elastic billiard ball, Echeverria and Klinkhammer[13] have identified sets of initial data with nonzero measure that can give rise to more than one self-consistent evolution. Figure 7 shows a simple example (due originally to Thorne): The evolutions in diagrams (a) and (b) are two self-consistent outcomes from the same initial data.

The initial data for Fig. 7 are posed before the Cauchy horizon of Fig. 3(a), but Fig. 7 shows the evolution only long after that horizon, when the second wormhole mouth has come to rest. The initial data consist of a single billiard ball moving along a trajectory that is aimed half way between the two wormhole mouths (trajectory α). In evolution (a) [henceforth we shall call it *classical history* (a)] the ball moves freely between the mouths, never colliding with anything. In classical history (b) the ball gets hit by itself and knocked down mouth 2 at point A, whereupon it emerges from mouth 1 (earlier in external time) moving along trajectory β, and hits itself. Echeverria and Klinkhammer[13] have shown that the existence of two classical histories, type (a) and type (b), is stable against small perturbations of the initial data, and have identified a variety of other sets of initial data with multiple classical histories.

H. Quantum effects: The billiard ball problems

This multiplicity of solutions is disturbing. However, it appears to be an artifact of classical physics that goes away in a fully self-consistent, quantum mechanical treatment.

The most attractive approach to quantum theory in the presence of closed timelike curves (but perhaps not the only viable approach) is Feynman's sum over histories.

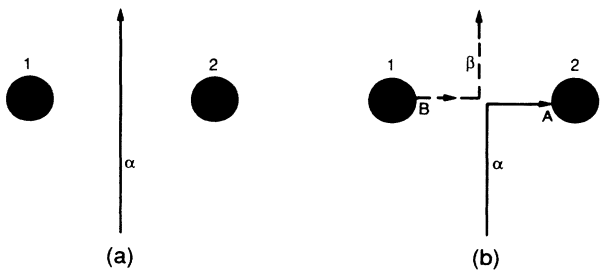


FIG. 7. Two classical evolutions (histories) for a billiard ball that result from the same initial data (position and velocity).

One of the authors (Friedman[25]) has previously advocated this view. The sum over histories meshes especially nicely with the principle of self-consistency. To impose that principle one need only ensure that the sum includes every self-consistent history, and only self-consistent ones. Two of the authors (Klinkhammer and Thorne[14]) have developed such a sum-over-self-consistent-histories formulation of nonrelativistic quantum mechanics for the self-interacting billiard ball and have found, not surprisingly, that it gives a unique, self-consistent set of probabilities for the outcomes of all sets of measurements one might imagine making; i.e., it removes the classical theory's multiplicity of solutions—or, at least this is so if the sum over histories converges. (No attempt has been made to prove convergence.)

When the billiard ball begins in a nearly classical wave packet corresponding to the initial conditions of Fig. 7, a WKB approximation to the sum over histories predicts a 50% probability for the ball's wave packet to emerge from the wormhole region moving along each of the two classical trajectories, Fig. 7(a) and Fig. 7(b).[14]

When the ball begins in a nearly classical wave packet for which there are no self-consistent classical evolutions, the WKB approximation must fail, and presumably (though no proof of this has been given) the existence of CTC's will cause the probabilities of finding the ball at various locations to get smeared out rather than remain highly localized.

III. 2-DIMENSIONAL, FLAT, TOROIDAL SPACETIMES

The two most important features of the Cauchy problem for a classical massless scalar field Φ in the wormhole spacetimes of Sec. II are (i) the fact that initial data posed before the Cauchy horizon are unconstrained by the principle of self-consistency; and (ii) the fact that initial data posed in the region with CTC's are locally unconstrained—i.e., these spacetimes are benign with respect to Φ—at least in the geometric optics limit.

While we suspect that these properties hold also in other generic, asymptotically flat, 4-dimensional spacetimes, they certainly do not hold in all spacetimes—or, at least, the benign property does not. Flat, toroidal spacetimes provide simple counterexamples, which we shall study in this section:

The spacetimes we shall study all have the flat, 2-dimensional metric

$$ds^2 = -dT^2 + dX^2 , \tag{13a}$$

where T is periodic with period P:

$$(T,X) \text{ is the same event as } (T+P,X) . \tag{13b}$$

Such a spacetime is closed in the time direction. We shall study the initial value problem for a classical, massless scalar field Φ in such a spacetime for the cases where X runs from $-\infty$ to $+\infty$ (Sec. III A), and where X is periodic with period L, and P/L is an integer, or is rational, or is irrational (Sec. III B).

A. Spatially infinite spacetime

In the spatially infinite case the easiest place to pose initial data for $\Box\Phi=-\Phi_{,TT}+\Phi_{,XX}=0$ is on a timelike surface of constant X rather than a spacelike surface of constant T. Because the wave equation is insensitive to which of the two directions is time and which is space, it should be clear that a complete set of initial data on, say, $X=0$, is $\Psi_1(T)\equiv\Phi(T,0)$ and $\Psi_2(T)\equiv\Phi_{,X}(T,0)$, where both Ψ_1 and Ψ_2 are periodic in T with period P. From these two functions one can construct a unique solution to $\Box\Phi=0$:

$$\Phi(T,X)=\frac{1}{2}[\Psi_1(T-X)+\Psi_1(T+X)]+\frac{1}{2}\int_{T-X}^{T+X}\Psi_2(s)ds\ . \tag{14}$$

This general solution is made more clear by splitting Ψ_2 into a constant piece (call it C) plus a piece whose integral over the time period P vanishes:

$$\Psi_2(T)=C+\tilde{\Psi}_2(T)\ ,\quad \int_0^P\tilde{\Psi}_2(T)dT=0\ . \tag{15}$$

Then the solution (14) becomes

$$\Phi(T,X)=\frac{1}{2}[\Psi_1(T-X)+\Psi_1(T+X)]+\frac{1}{2}\int_{T-X}^{T+X}\tilde{\Psi}_2(s)ds+CX\ . \tag{16}$$

It is easy to see that the pieces generated by Ψ_1 and $\tilde{\Psi}_2$ are strictly periodic in space X with period P (as well as periodic in T with period P), while the CX is monotonic increasing in X.

Knowing this general solution to $\Box\Phi=0$, we can turn it around and deduce the free initial data on a spacelike surface of constant time, say $T=0$. It should be clear that those free data are Φ and $\Phi_{,T}$ subject, however, to the following self-consistency constraints: (i) $\Phi_{,T}(X)$ must be periodic in X with period P and its integral over any interval (X_0,X_0+P) of length P must vanish; and (ii) $\Phi(X)$ must consist of a piece that is periodic with period P, plus a piece that is linear in X. These are relatively mild constraints; mild enough, in fact, to demonstrate that the spacetime is benign with respect to the field Φ. We shall call these two constraints *P periodicity*.

B. Spatially closed spacetime

If the spacetime of Eqs. (13) is spatially periodic with period L, this periodicity will change the character of the initial-value problem. In particular, it will force the initial data $\Phi_{,T}(X)$ and $\Phi(X)$ at $T=0$ to be periodic with period L. We shall call this constraint, together with an additional integral constraint, $\int_{X_0}^{X_0+L}\Phi_{,T}(X)dX=0$ for all X_0, *strict L periodicity*.

If the ratio P/L is an integer, then strict L periodicity implies P periodicity with the term linear in X vanishing, i.e., implies strict P periodicity. Stated more precisely, if P/L is an integer, then the existence of CTC's places only one minor new constraint on the initial data, beyond those required by spatial closure: the constraint that the integral of $\Phi_{,T}(X)$ over any interval of length P must vanish. Of course, in this case as for the spatially infinite case, the spacetime is benign for Φ.

If the ratio P/L is rational but nonintegral, then strict L periodicity does not imply P periodicity. However, strict L periodicity and P periodicity are perfectly compatible: they are both guaranteed by the demand that the initial data $\Phi_{,T}(X)$ and $\Phi(X)$ be strictly S periodic (no linear term), where S is the largest number such that P/S and L/S are both integers. This constraint on the initial data leaves the spacetime benign for Φ.

Most interesting is the case where P/L is irrational. In this case P periodicity and strict L periodicity are mutually incompatible; and, correspondingly, *there are no self-consistent solutions to* $\Box\Phi=0$ except constants. This means that the spacetime is not benign. Moreover, with the obvious measure put on the class of toroidal 2-dimensional flat spacetimes of the above type [metric (13) with periodicity in both T and X], all but a set of measure zero have P/L irrational. This means that all but a set of measure zero fail to be benign. A similar but slightly more involved analysis by one of the authors (Yurtsever[12]) shows that this is true much more generally in two dimensions: Among *all* two-dimensional compact spacetimes, those that are benign constitute a subset of measure zero.

We suspect that the opposite is true for realistic, four-dimensional wormhole spacetimes: All but a set of measure zero will turn out to be benign. If so, and if the laws of physics turn out to permit traversible wormholes with stable Cauchy horizons, then theoretical physics might adjust rather easily to the CTC's that wormhole spacetimes produce.

A much more detailed discussion of the notion of benignness in compact spacetimes (including the interconnections between benignness and the topology and geometry of the spacetime) are given in Ref. 12.

IV. CONCLUSIONS AND SPECULATIONS

We conclude with a few remarks and speculations.

Most importantly, we wish to reemphasize that it is far from obvious whether or not the laws of physics permit CTC's.[1] If it turns out that CTC's are forbidden, then this paper will become a somewhat irrelevant exercise. If CTC's are allowed, then the considerations in this paper may point the way toward an accommodation between them and theoretical physics. Such an accommodation, we hope, would someday entail proofs that the properties we see hinted at in our model wormhole spacetimes and for our model problems are true quite generally: (i) the CTC's allowed by physical law leave spacetime benign, (ii) they leave unconstrained, at least in quantum theory if not in classical theory, those initial data which are posed before the region with CTC's, and (iii) they leave unique the evolution of probability amplitudes as dictated by the path-integral approach to quantum mechanics.

If CTC's are allowed, and if the above vision of theoretical physics' accommodation with them turns out to be more or less correct, then what will this imply about the philosophical notion of free will for humans and other intelligent beings? It certainly will imply that

intelligent beings cannot change the past. Such change is incompatible with the principle of self-consistency. Consequently, any being who went through a wormhole and tried to change the past would be prevented by physical law from making the change; i.e., the "free will" of the being would be constrained. Although this constraint has a more global character than constraints on free will that follow from the standard, local laws of physics, it is not obvious to us that this constraint is more severe than those imposed by standard physical law.

It is tempting to speculate about possible connections, in spacetimes with CTC's, between free will and the Everitt-Wheeler many-worlds interpretation of quantum mechanics. However, we shall leave such speculations to the reader.

ACKNOWLEDGMENTS

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APPENDIX: STRUCTURE OF THE CAUCHY HORIZON

In this appendix we elucidate the structure of the Cauchy horizon $\mathcal{H}$ in the wormhole spacetime of Fig. 3(a).

The details of the horizon's structure depend on the details of the mapping between the two wormhole mouths, i.e., on which points are connected to which through the infinitesimally thin throat of the wormhole. Our choice of mapping is this (Fig. 8): During the epoch when the Cauchy horizon occurs, the left mouth is at rest on the spatial origin, while the right mouth moves toward it along the X axis [Fig. 3(a)]. Set up a right-handed spherical polar coordinate system (θ,ϕ) on the right mouth with polar axis pointed in the $-X$ direction, i.e., toward the left mouth, and with $\phi=0$ in the X-Z plane; and set up a left-hand spherical polar coordinate system (θ,ϕ) on the left mouth with polar axis pointed in the $+X$ direction, i.e., toward the right mouth, and with $\phi=0$ in the X-Z plane. Then points on the two mouths with the same values of θ and ϕ are identified.

The details of the horizon's structure are obscured by the text's idealizing the wormhole throat as infinitesimally short. Accordingly, in this appendix we shall give the throat a finite but very small thickness, as shown in the embedding diagrams of Fig. 9.

In any asymptotically flat spacetime, any Cauchy horizon $\mathcal{H}$ that separates a past region without CTC's from a future region with them has several important geometric properties.

(1) The Cauchy horizon is a null surface generated by null geodesics with no past endpoints. [See Proposition 6.5.3 of Ref. 26, in which the set S can be any spacelike hypersurface that precedes the Cauchy horizon and extends to spacelike infinity. This S is a closed achronal set without edge, and $H^+(S)$ is our Cauchy horizon $\mathcal{H}$. According to Proposition 6.5.3, the generators must continue into the past indefinitely, as null geodesics and generators, without intersecting the hypersurface S.]

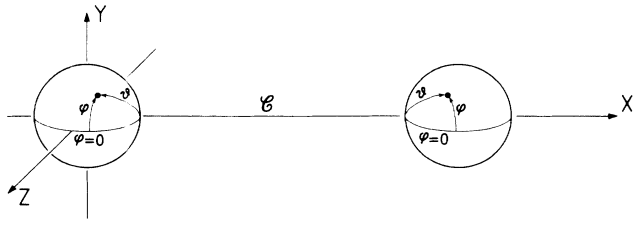


FIG. 8. Spatial diagram showing how points on the two wormhole mouths are connected to each other. Points with the same values of θ and ϕ are the same.

(2) No two events on the Cauchy horizon $\mathcal{H}$ can be joined by a timelike curve. (Suppose that event $\mathcal{A}$ on $\mathcal{H}$ were connected to event $\mathcal{B}$ on $\mathcal{H}$ by a future-directed timelike curve $\mathcal{K}$. Since $\mathcal{H}$ is null, $\mathcal{K}$ can cross it in only one direction: from the region without CTC's to the region with them. Thus, $\mathcal{K}$ must enter the region with

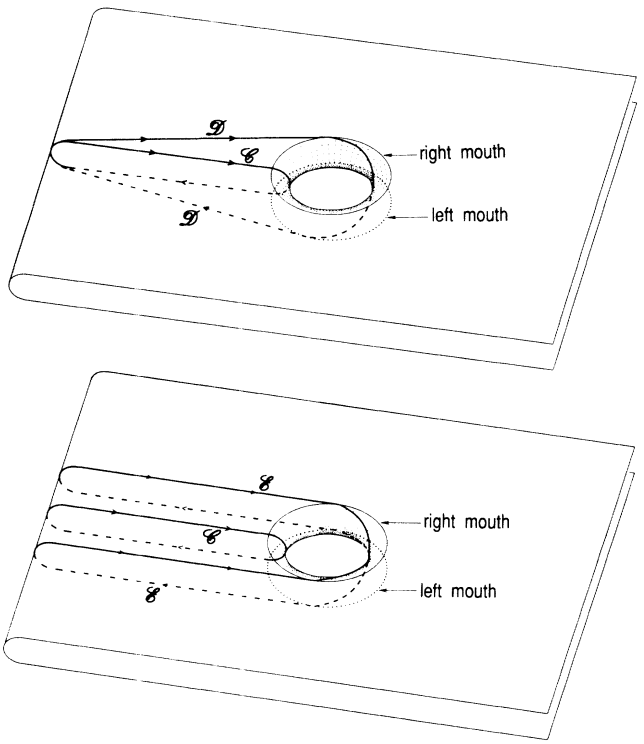


FIG. 9. Embedding diagrams depicting the spatial geometry of the wormhole, when the throat is given a finite but tiny thickness. These diagrams are for a slice of simultaneity as measured by observers at rest relative to the wormhole nearest to them. The fact that the mouths move toward each other shows up in a nonmeshing of the slice onto itself in spacetime halfway between the mouths, i.e., at the bend at the left edge of the diagram. If the relative speed of the mouths is small compared to the speed of light (the case shown here), this nonmeshing is tiny; if the relative speed is close to that of light, the nonmeshing becomes significant.

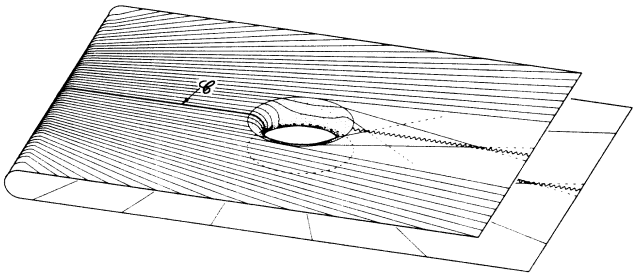


FIG. 10. Spatial depiction of the null geodesic generators of the Cauchy horizon. All the generators peel off the closed null curve $\mathcal{C}$. Notice the two caustics, which are shown as jagged lines.

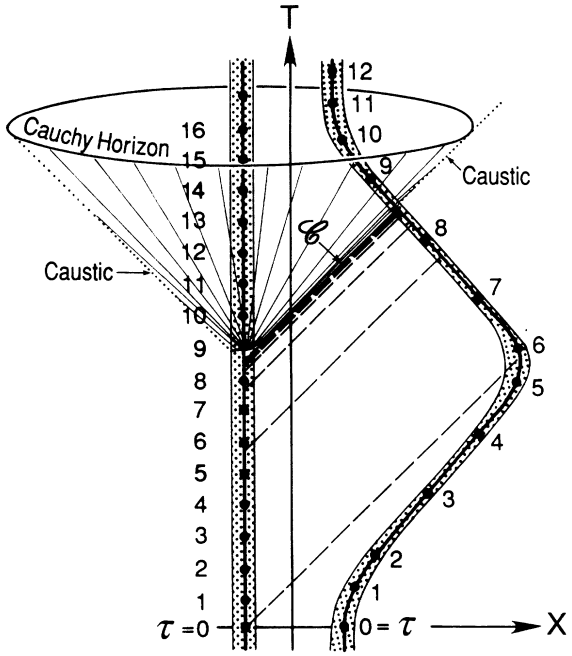


FIG. 11. Spacetime depiction of the Cauchy horizon. The stippled region is excised from the spacetime and its outer edges are joined together to make the wormhole throat. Along the stippled region is indicated proper time as measured in the wormhole throat. Notice the two caustics on the Cauchy horizon.

CTC's at its starting point $\mathcal{A}$, it cannot ever thereafter leave that region, and it therefore must arrive at $\mathcal{B}$ from the side with CTC's. However, this means that, if prolonged slightly beyond $\mathcal{B}$, $\mathcal{H}$ will pass in a future-directed manner from the region with CTC's to the region without, which is impossible.)

(3) No two generators can cross each other when followed into the past, and a generator cannot cross itself. [The finite crossing angle in spacetime would permit points on one generator to be joined to points on the other generator by timelike curves, in violation of property (2).]

These three properties permit us to deduce the structure of the Cauchy horizon $\mathcal{H}$ for the spacetime of Fig. 3(a). Because this spacetime is axisymmetric about the line of centers between the wormhole mouths with hypersurface-orthogonal Killing vector $\partial/\partial\phi$, its Cauchy horizon $\mathcal{H}$ will be axially symmetric and the generators of $\mathcal{H}$ will lie in poloidal 3-surfaces of constant ϕ (where ϕ is as defined in Fig. 8). This means we can restrict attention to a representative such 3-surface, say that with $\phi = 0$, the two spatial dimensions of which are depicted by the embedding diagrams of Fig. 9. The only way that null geodesic generators, lying in this 3-surface, can extend indefinitely into the past while never crossing each other or themselves and never developing timelike separations [i.e., the only way they can satisfy the properties (1), (2), and (3)] is by asymptotically approaching one or more closed null geodesics (CNG). Figure 9 shows the spatial trajectories of three such CNG's: curves $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{E}$. With these examples of CNG's as prototypes, one can readily identify others.

The CNG $\mathcal{E}$ cannot be an asymptote of generators of $\mathcal{H}$ because it crosses itself. The two curves $\mathcal{C}$ and $\mathcal{D}$ cannot both be asymptotes because they either cross each other or have events that are timelike separated. [The event where $\mathcal{D}$ crosses the X axis lies at the same location in space as some event on $\mathcal{C}$ (as seen in any Lorentz frame that moves along the X-axis); and those two events can thus be connected by a timelike curve or are the same.] Since the spatial curve $\mathcal{C}$ drawn in Fig. 9(a) has a shorter spatial length than $\mathcal{D}$, as the two wormhole mouths move toward each other, it will become null (and thus become the CNG $\mathcal{C}$) sooner than does $\mathcal{D}$. Thus, the CNG $\mathcal{C}$ lies to the past of the CNG $\mathcal{D}$, which means that $\mathcal{C}$ is the candidate for the asymptote of the null generators, not $\mathcal{D}$.

By extending this type of argument to the other CNG's, one can show that $\mathcal{C}$ is, indeed, the asymptote; i.e., all null generators of $\mathcal{H}$ emerge from (peel off) $\mathcal{C}$ when followed into the future.

Figures 10 and 11 depict, in a spatial embedding diagram and in spacetime, the null geodesic generators and the Cauchy horizon they generate. Notice that the Cauchy horizon $\mathcal{H}$ possesses two caustics at which its null generators leave $\mathcal{H}$ when followed toward the future. One caustic lies on the X-axis to the right of the right mouth; the other, on the X-axis to the left of the left mouth. Aside from these two caustics the Cauchy horizon is everywhere smooth.

For the spacetime of Fig. 3(b), which is not axisymmetric and whose Cauchy horizon $\mathcal{H}_-$ we have not studied in detail, there might be two or more spacelike separated CNG's which act as asymptotes for the generators of $\mathcal{H}_-$. At least, this might be the case for a sufficiently large relative velocity of the two mouths. However, when the relative velocity is sufficiently small, one of these CNG's, $\mathcal{C}_-$, will have a shorter spatial length than the others and thus will occur earlier in time than the others — sufficiently earlier that the others all lie in its future, thereby leaving $\mathcal{C}_-$ as the sole asymptote for the generators of $\mathcal{H}_-$.

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[5]Previously we had thought (Ref. 1) that quantum field theory might generically impose the averaged weak energy condition, independently of the geometry or topology of the spacetime in which the quantum fields reside. However, it now appears that such enforcement, if it occurs at all, is spacetime dependent: G. Klinkhammer (paper in preparation) has shown that for a scalar field in Minkowskii spacetime the averaged weak energy condition is always satisfied in sufficiently well-behaved quantum states (probably including all physically realistic states), but that when Minkowskii spacetime with Lorentz coordinates (t, x, y, z) is made toroidal (cylindrical) by identifying the 3-surface $x=0$ with $x=L$, and the scalar field is put into its Casimir-like vacuum state, then the averaged weak energy condition is violated along null geodesics that travel in the x direction.

[6]Violation of the averaged weak energy condition is, of course, a necessity, but may not be a sufficient, requirement for traversible wormhole maintenance. See, e.g., L.H. Ford and T.A. Roman, Phys. Rev. D **41**, 3662 (1990), where explicit calculation suggests that some sort of "quantum inequality" might be enforced by quantum field theory on curved spacetime. The connection of this kind of energy condition with the averaged weak energy condition is at present unexplored.

[7]See, e.g., S.W. Hawking, Nucl. Phys. **B244**, 135 (1984); S.W. Hawking, D.N. Page, and C.N. Pope, *ibid.* **B170**, 283 (1980), and references to earlier work cited therein. Subsequent papers on Euclidean wormholes by S.W. Hawking, Phys. Rev. D **37**, 904 (1988); S. Coleman, Nucl. Phys. **B307**, 864 (1988); S. Giddings and A. Strominger, *ibid.* **B307**, 864 (1988); H.E. Kandrup and P.O. Mazur, Mod. Phys. Lett. A **4**, 1189 (1989); R.C. Myers, Phys. Rev. D **38**, 1327 (1988), were followed by a large number of articles. References to these can be found in W. Unruh, Phys. Rev. D **40**, 1053 (1988); W. Fischler, I. Klebanov, J. Polchinski, and L. Susskind, Nucl. Phys. **B327**, 157 (1989); J.J. Halliwell and R.C. Myers, Phys. Rev. D **40**, 4011 (1989); J. P. Preskill, Nucl. Phys. **B323**, 121 (1989). Also see J.J. Halliwell, Int. J. Mod. Phys. A **5**, 2473 (1990).

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[10]I. D. Novikov, *Evolution of the Universe* (Cambridge University Press, Cambridge, England, 1983), p. 169. Ya. B. Zel'dovich and I. D. Novikov, *Stroeniye i Evolyutsia Vselennoi* (Nauka, Moscow, 1975), p. 679. Note that in the English edition of the latter reference [*Relativistic Astrophysics, Vol. 2, The Structure and Evolution of the Universe* (University of Chicago Press, Chicago, 1983), bottom of page 637 and top of page 638] the text was changed without Novikov's agreement to say that CTC's cannot occur.

[11]J. L. Friedman and M. Morris (unpublished).

[12]U. Yurtsever, J. Math. Phys. (to be published).

[13]F. Echeverria and G. Klinkhammer (unpublished).

[14]G. Klinkhammer and K. S. Thorne (unpublished).

[15]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[16]M. Morris and K. S. Thorne, Am. J. Phys., **56**, 395 (1988).

[17]J. L. Friedman, L. Parker, N. J. Papastamatiou, and H. Zhang, Nucl. Phys. **B309**, 533 (1988).

[18]The nature of those errors can be explored easily, in the special case of a wormhole with one mouth unaccelerated and the other uniformly accelerated. For this purpose, we introduce coordinates t, l, θ, ϕ, that extend through the wormhole's throat ($l=0$) from the unaccelerated side $l<0$ where they are inertial, to the accelerated side $l>0$, where they are uniformly accelerated. The metric in this coordinate system takes the form (Ref. 1)

$$ds^2=-(1+gFr\cos\theta)^2dt^2+dl^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\ .$$

Here $r=b+|l|$; and $F=0$ at $l<0$ and F rises smoothly from 0 to unity as l increases from 0 to some arbitrarily small value ϵ—i.e., F is a slightly smoothed step function. In this coordinate system the scalar wave equation takes the form $\Box_s\Phi=-gb\cos\theta F_{,l}\Phi_{,l}$, where $\Box_s$ is the part of the scalar wave operator that does not depend on the derivative of F, the right-hand side is the dominant effect of the acceleration g, and we have ignored fractional corrections of order gb to the right-hand side. Since the highest radial derivatives that appear in $\Box_s$ are $\partial^2/\partial l^2$ with no coefficient in front, the dominant effect of the acceleration is to produce, over the tiny region $0\le l\le\epsilon$, a jump in the logarithm of $\Phi_{,l}$: $\Delta(\ln\Phi_{,l})=-gb\cos\theta$. Correspondingly, there will be a fractional jump $-gb\cos\theta$ in the wavelengths of short-wavelength ($\lambda\ll b$ but $\lambda\gg\epsilon$) waves that propagate through the wormhole's throat. This is the dominant error ignored by the text's discussion.

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